In the paper the method for estimation of settlement of vertically loaded piles in a group is presented. The method is generally based on the hybrid procedure with non-linear solution for single piles and with interaction between them for the low-strain shear modulus. Some attention is paid to the influence of deformation and strength parameters used in calculations.

The applied model is verified in the framework of the research programme carried out recently in the Geotechnical Department of Gdansk Technical University. It concerns monitoring and analysing the settlements of the existing structures on piles. A comparison of calculation and measurements results for two tested buildings is presented as an example.

INTRODUCTION

Presently, piles and pile foundations are not only being applied in the case of a subsoil built of weak soils but are also used in relatively good soil conditions in order to assure safe transmission of stresses into the subsoil. The later mostly regards such areas as industrial and bridge construction and hydro-engineering. In spite of quick development of new techniques of pile installation and more reliable calculation methods supporting the design process there still exists a lack of appropriate theoretical approach for the problem of the work of pile groups.

For the same loads per one pile and at the same soil conditions the settlement of the group of piles is much larger than for a single pile. It mainly results from the mutual penetration and overlapping of stress zones of particular piles and essentially bigger area of stress interaction below the pile group. The settlement of the pile group may reach a magnitude of several centimetres. It depends on soil conditions, foundation dimensions, number of piles, its spacing, the method of installation etc.

The settlement of the pile group depends on several phenomena occurring during transmission of loads into the subsoil. Only some of them are already defined and elaborated theoretically. Creation of new tools supporting the design of piling foundations after all requires a good recognition and understanding of these processes.

Main source of the knowledge about the phenomena occurring within the interaction zone between piles and soil medium are observations of structures founded on piles, results of model tests and calculation analyses based on the theoretical assumptions and making use of numerical codes.

Serious difficulties in the modelling of factual pile work in the subsoil restrict the use of model tests to the qualitative analysis of the phenomena investigated, only.

The quantitative analysis can be carried out using field observations of existing and currently constructed structures on piles supported by analytical verification of phenomena analysed. This problem is a subject of research work, which is currently carried out in Geotechnical Department of Gdansk Technical University. The basic objective of the research is the detailed analysis and comparison of existing calculation methods for the estimation of the settlement of pile in-group and the analysis of actual behaviour of piling foundations. Tejchman et al. (2001) have presented the details of the research with some results and calculation analyses.

Detailed analysis of the results of measurement and calculations has enabled accurate verification of existing calculation methods and contributed to the elaboration of the method for calculation of the settlements of pile group subjected to vertical forces.

METHOD FOR CALCULATION OF THE SETTLEMENTS OF PILES IN GROUP

Methods for calculation of the settlements of piling groups can be found elsewhere, (c.f. El Mossalamy and Franke, 1997; Randolph, 1994; Van Impe, 1991). The analyses performed show that interaction between piles calculated at the assumption that piles work in linearly elastic soil medium significantly overestimate the settlements (see Tejchman at al., 2001). The magnitudes of settlements achieved on the basis of precise levelling method are usually 3 to 5 times lower (on the average) than the values calculated by analytical methods. In the group of analytical methods the influence coefficient method by Poulos (1980) is also considered. This method is included in Polish Piling Code (Gwizdala, 1997). Promising results, close to the measured values, one can obtain in terms of equivalent raft foundation method. However the result is equivalent only to the mean value of foundation settlements and
estimation of the settlements of particular piles in the group is not possible.

Main reason of the achieved overestimated values of the settlements in the analyses of pile-subsoil interaction is the assumption of the soil as linearly elastic medium, independent of the stress state.

The work of the piles in the group is of non-linear character. This non-linearity is caused by both a characteristics of the subsoil work as well as the phenomena occurring within the contact zone between the pile and the soil. Due to exceeding of the ultimate resistance, the upper sectors of the pile can be excluded from the work already in the initial stress state and the mobilisation of the resistance of the pile’s shaft is quickly transferred to the base. It causes an increase of loads of lower parts of the pile together with its base, where the interaction between the piles is smaller than in the upper parts. Additionally, one can observe a change of shear modulus of the soil with the change of stress-strain state and with increasing depth (Van Impe and De Clercq, 1994; Gwizdala and Tejchman, 1997). This variability should be considered at least in the zone of contact between the pile and the soil. However, due to small deformations, the interaction between the piles should be still treated as for linearly elastic medium.

The presented method is generally based on the hybrid approach according to Chow (1986) with non-linear solution for single piles and with interaction between them for low-strain shear modulus.

In order to reflect the non-linear character of pile work prior the achievement of limit resistance within the contact zone between the pile and the soil a non-linear plastic model has been applied.

Non-linear characteristic of pile work has been described by hyperbolic function representing the change of shear modulus as a function of mobilised soil resistance, eq. 1. A constant $R_f$ determines the shape of the curve. For the pile base, where the non-linearity is more pronounceable, the value of $R_f = 0.9$ was assumed whereas for the pile shaft $R_f = 0.5$ (Fig. 1). Respective calculation results can be found in the work of Dyka (2001).

$$G = G_{\text{max}} \left( 1 - \frac{R}{R_f} \right)^2, \quad (1)$$

where:

- $G_{\text{max}}$ – initial shear modulus,
- $R$ – mobilised soil resistance,
- $R_f$ – limit soil resistance (at failure),
- $R_f$ – constant of hyperbolic function.

![Figure 1. Mechanical characteristic for a single pile.](image)

Fig. 1. Mechanical characteristic for a single pile.

For pile’s shaft the load-transfer functions are based on the solution proposed by Randolph and Wroth (1978):

$$s = \frac{\tau R_0}{G_s} \ln \left( \frac{R_{\text{max}}}{R_0} \right) \quad (2)$$

where:

- $s$ – displacement of the point of pile’s shaft,
- $\tau_0$ – shear stress along the pile’s shaft,
- $R_0$ – radius of the pile
- $R_{\text{max}}$ – the range of the pile interaction,
- $G_s$ – shear modulus of the soil along the pile’s shaft.

Maximum radius of pile interaction – $R_{\text{max}}$, is calculated according to the formula proposed by Van Impe and De Clercq (1994) who assume its change in a function of depth:

$$R_{\text{max}} = 2(1-\nu) L \left( \frac{3}{2} - \frac{z}{L} \right) \quad (3)$$

where:

- $L$ – length of the pile in the soil,
- $z$ – depth,
- $\nu$ – Poisson’s ratio.

![Figure 2. Comparison of the changes of shear modulus in a function of shear strain.](image)

Fig. 2 shows the comparison of this solution with the solutions obtained and presented by other authors.

Making use of numerical codes the pile, being treated as elastic medium and described by Young’s modulus, is digitised by small elements with elastic supports in its nodes (see Fig. 3). The characteristic of the supports is determined by load-transfer function. For pile in-group, additional movement caused by interaction of other piles is determined using Mindlin’s solution for the force acting inside uniform, isotropic, elastic half-space.

![Figure 3. Load-transfer functions.](image)

For pile’s shaft the load-transfer functions are based on the solution proposed by Randolph and Wroth (1978):
The base of the pile acts as rigid punch on the subsoil surface. The displacement of the base $s_b$ under subjected force $P_b$ can be described by Boussinesq formula:

$$s_b = \frac{P_b}{R_b G_b} \left(1 - \nu_s\right) \mu_d$$  \hspace{1cm} (4)

The coefficient $\mu_d$ in Eq. (4) reflects the influence of the base depth. For practical ratios of the pile diameter to the depth the coefficient has been assumed on the basis of the curve proposed by Fox, i.e. $\mu_d = 0.5$.

The value of shear modulus decreases in the non-linear way according to the function assumed:

- along the pile's shaft:
  $$G_s(\tau) = G_{\text{max}} \left(1 - \frac{\tau \cdot R_t}{r_f}\right)^2$$  \hspace{1cm} (5)

- under the pile base:
  $$G_b(q) = G_{\text{max}} \left(1 - \frac{q \cdot R_t}{q_f}\right)^2$$  \hspace{1cm} (6)

where:
- $G_s(\tau)$, $G_b(q)$ - current, tangent shear modulus for the shaft and base of pile, respectively,
- $G_{\text{max}}$ - initial shear modulus for small deformations,
- $\tau$ - current shear stress along the pile's shaft,
- $q$ - current normal stress under the pile base,
- $R_t$ - constant for hyperbolic curve,
- $\sigma$ - limit shear resistance - shear strength,
- $q_f$ - limit resistance under the base of pile.

Interaction between particular piles in the group is treated as linearly elastic and is described by shear modulus $G_{\text{max}}$ in Mindlin's solution.

Particular layers of the soil are described by following parameters: shear modulus, Poisson's ratio, and limit shear resistance and limit resistance under the base of a pile.

After division of the piles' system onto small elements the rigidity matrix is constructed. The dimension of the matrix $[N]$ corresponds to a total number of nodes of whole piles in the group. The settlements of particular nodes are a solution of global set of equations:

$$[Q] = [K][s],$$  \hspace{1cm} (7)

where rigidity matrix $[K]$ is a sum of respective matrices of the soil and the piles : $[K] = [K_s] + [K_p]$. $\{Q\}$ denotes vector of external loads of system nodes and $\{s\}$ is the vector of searched nodal settlements.

The calculations are conducted for following load increments. Next new value of soil modulus is calculated in particular step for current mobilised resistance in particular node.

An increase of load causes more intensive mobilisation of the soil resistance. When the ultimate resistance is achieved in any of nodes, then slip occurs there, what results in excluding this node for subsequent load increments. It is reflected by a stabilisation of the soil response for a given node (the soil resistance does not increase in this place) together with the lack of its interaction with the neighbours.

The numerical procedure described enables the determination of load-settlement curve for a single pile as well as for an arbitrary pile in the group for wide range of loads, complex soil conditions and arbitrary system of piles. Additionally, the results obtained allow the decomposition of the total load onto the load transmitted by shaft and base of the pile together with redistribution of loads with depth for specified load increments.

**SURVEY MEASUREMENTS OF THE FOUNDATION DISPLACEMENTS**

According to the research programme several existing structures such as silos, large tanks, flyovers, bridges and buildings have been subjected to monitoring by survey measurements of the foundation settlements. The structures were founded on different types of piles, with different spacing and dimensions of the foundations and different soil conditions. The survey measurements were performed in terms of precise levelling.

The investigations have started during initial stage of the construction (after concreting of the foundations) and have been continued for many years covering the construction phase and later exploitation of the structure. The measurements allowed the observation...
of the progress of settlements during increasing loads and also the comparison of the results with the predictions obtained by various calculation methods.

In the analysis all data concerning construction details of foundation and superstructure, its geometry, piles' characteristics, geotechnical conditions (geotechnical profiles, CPT, SPT, DPS tests etc.) together with the results of load tests performed on single piles.

**Ash tanks in Thermal-Electric Power Station**

This is the foundation for two cylindrical tanks in T-E P S in Gdansk, Poland. The foundations are loaded by two tanks to store the ashes. The height of each tank is 24 m and the diameter 12 m.

The tanks are resting on eight columns, which transmit the loads onto foundations.

![Figure 4. Ash tanks of T-E P S. Layout of the pile foundation.](image)

Dimensions of foundations are the following:
- 4 foundations: $B \times L = 4.3 \times 4.5$ m; $H = 2$ m;
- 2 foundations: $B \times L = 4.3 \times 10.0$ m; $H = 2$ m.

**Silos of Melt Factory (M F)**

The object is localised at Bytom Quay in Gdansk Port. The analysis regards foundation under battery of 12 reinforced concrete silos of circular cross-section and arranged into two rows (Fig. 6). The load from bearing walls is transmitted onto 22 piles under each silo by a slab 17.5 m wide, 51.0 m long and 0.5 m thick.

Totally, 264 “Vibrex” piles of the shaft diameter of $D = 508$ mm and base diameter of $D_b = 620$ mm and 13.5 m long have been installed (Fig. 7).

The piles have been embedded into well-compacted sandy layer with $I_D = 0.85$. Groundwater table locates approximately at 1.50 m a.s.l.

Characteristic value of top load per single pile was $Q = 880$ kN.

For control purposes in foundation slab totally 16 benchmarks have been installed, Fig. 6.

![Figure 6. Silos of M F. Layout of the foundation.](image)

![Figure 7. Silos of M F. Arrangement of piles in the foundation.](image)
RESULTS OF SETTLEMENT CALCULATIONS AND MEASUREMENTS

For all monitored objects the calculations by various methods (including the method presented above) have been carried out. The analysis of the results shows an essential difference of the magnitude of settlements between various calculation methods (see Table 1).

Table 1. The results of measurements and calculations.

<table>
<thead>
<tr>
<th>Object</th>
<th>T-E P S Tanks</th>
<th>M F Silos</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method</td>
<td>Settlement [mm]</td>
<td></td>
</tr>
<tr>
<td>Equivalent raft foundation acc. to PN-83/B-02482 (1983)</td>
<td>15,1</td>
<td>35,3</td>
</tr>
<tr>
<td>Equivalent raft foundation acc. to Tomlinson (1994)</td>
<td>15,8</td>
<td>40,7</td>
</tr>
<tr>
<td>Equivalent raft foundation acc. to Van Impe (1991)</td>
<td>6,2</td>
<td>35,5</td>
</tr>
<tr>
<td>Equivalent column acc. to Poulos (1980)</td>
<td>calc. by Randolph's equation (1994)</td>
<td>22,6</td>
</tr>
<tr>
<td>Influence coefficient method acc. to Poulos (1980)</td>
<td>calc. by Chow's approach (1986)</td>
<td>26,0</td>
</tr>
<tr>
<td>Chow's approach (1986)</td>
<td>25,9</td>
<td>92,5</td>
</tr>
<tr>
<td>Randolph's equation (1994)</td>
<td>30,7</td>
<td>96,6</td>
</tr>
<tr>
<td>Method proposed by the authors</td>
<td>8,5</td>
<td>20,9</td>
</tr>
<tr>
<td>Measurement</td>
<td>7,0</td>
<td>17,0</td>
</tr>
</tbody>
</table>

Figs. 9 – 12 present the results of settlement measurements of mentioned constructions together with the results of calculations obtained in terms of proposed method. The conformity of the predicted Q-s curves with those obtained from load tests of single piles is satisfying (Figs. 9 and 11).

The longitudinal profiles of measured and calculated settlements are shown in Figs. 10 and 12. Also in this case the conformity of the settlement magnitudes as well as of its distribution along foundations is acceptable.
CONCLUSIONS

The comparison of the calculation results obtained by the proposed method with measured values shows that the method allows good prediction of the settlements for a single pile and pile foundation for wide range of loads, complex soil conditions and the arbitrary system of piles (see Table 2).

Table 2. Comparison of calculation results with measured values.

<table>
<thead>
<tr>
<th>Object</th>
<th>T-E P S</th>
<th>M F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of piles</td>
<td>72</td>
<td>264</td>
</tr>
<tr>
<td>Average settlement [mm]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calc. by proposed method</td>
<td>8,5</td>
<td>20,9</td>
</tr>
<tr>
<td>In-situ measured values</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pile group - se</td>
<td>7</td>
<td>17</td>
</tr>
<tr>
<td>Single pile - sp</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q = 953 kN</td>
<td>1,2</td>
<td></td>
</tr>
<tr>
<td>Q = 880 kN</td>
<td>1,0</td>
<td></td>
</tr>
</tbody>
</table>

The most important parameter in the calculations is related to the magnitude of deformation modulus, which vary with both stress and strain state. The methods based on the linear elastic medium assumption significantly overestimate the values of settlements of pile in-group. Mechanical characteristics of the work of pile in the soil are non-linear and therefore they can not be approximated by linear behaviour. However, the interaction between piles should be considered as for small deformations what means that respective moduli should correspond to the same assumption.

Shear modulus for small deformations is determined on the basis of the measurement of shear wave velocity in the soil. It may be done either in the triaxial apparatus equipped in bender elements, in resonant columns or in situ conditions making use of geophysical methods (cross-hole or down-hole tests). The another way is to use correlation with the results obtained by other tests e.g. CPTU and SEISMIC CONE. In the above analysis the initial value of shear modulus has been assumed G_{max} = 4G_{le}, where G_{le} denotes general shear modulus as for linearly elastic medium. In the case of the lack of appropriate tests, such a choice of parameters seems to be reasonable for the engineering point of view.

The analysis presented has confirmed the usefulness and effectiveness of the monitoring of foundation displacements during construction stage and in further long-term exploitation. The material collected is very good source of information regarding foundation-subsoil interaction and can be a basis for elaboration more rational calculation methods.

REFERENCES


